Fortunately, we have no rational use for flat roofs. Our cities are roomy, and our habits and their population will for many centuries keep them so. Our houses are low and our yards airy. I cannot conceive a single argument in favour either of the beauty or utility of terrace roofs in our country. Those that have them scarcely ever use them. The cold in winter and the heat in summer drive us from them. A beautiful prospect may justify the partial use of them, in particular situations, but neither architectural beauty, nor the general wants of our wintry climate call for their introduction.—To the southward beyond the reach of frost, however, the information contained in this paper may be highly useful.

No. LVII.

A general method of finding the roots of numeral equations to any degree of exactness; with the application of Logarithms to shorten the operation: by John Garnett of New Brunswick N. Jersey.

Read January 20th, 1809.

Suppose an equation, $ax+bx^2+cx^3+dx^4+ex^5$ &c. $=v$, to find x.

RULE.

Find, by trial, any near root as x'.

Then, by substitution, $ax'+bx'^2+cx'^3+dx'^4+ex'^5$ &c. $=v'$
Multiply each term by the index of the power of x', and divide by x'.
Let the products, $a+2bx'+3cx'^2+4dx^{3\prime}+5ex^4$, &c. $=A$.
Multiply each term by the power of x', and divide by $2x'$.
Let the products, $b+3cx'+6dx'^2+10ex'^3$, &c. $=B$.
Multiply each term by the power of x', and divide by $3x'$.
Let the products, $c+4dx'+10ex'^2$, &c. $=C$.
Multiply each term again by its power of x', and divide by $4x'$.
Let the products, $d+5ex'$, &c. $=D$: and so on, continually, until all the powers of x' are destroyed; so that e, &c. $=E$.

Then will $Ax''+Bx''^2+Cx''^3+Dx''^4+Ex''^5$ &c. $=v-v'$, be a *New Equation* whose roots will all be less by x', and the value, $v-v'$, less by v' than the roots of the original equation. And if the roots and value of this new equation be diminished in the same manner, by another near root, as x''', and so on, continually, the root and value may become less than any assignable quantity, and the sum of all the near roots will be equal to x, the root of the original equation.

This rule will be found virtually the same as that given by Newton, Raphson, Jones, Simpson &c. and, if applied to the extraction of simple powers, will be found the same precisely as the one usually given in common arithmetic, thus for

EXAMPLE I.

Let $x^3 =$	99252847		
$x' = 400$ - - - -	64000000	$= x'_3$	$x' = 400$
By the Rule, $v - v'$	35252847	Resolvend.	
divisor		New Equation $x'' = 60$	
$3x'^2 = 480000 = A$	28800000	$480000 \times x''$	
$3x' = 1200 = B$	4320000	$+1200 \times x''^2$	
$1 = 1 = C$	216000	$+1 \times x''3 = v - v'$	
By the Rule,	33336000	Subtrahend	
	01916847	Resolvend	

$A =$ divisor		New equation $x''' = 3$
$480000 + 2400x'' + 3x''^2 = 634800.$	$1904400 =$	$634800 \times x'''$
$B = 1200 + 3x = 1380.$	$12420 =$	$1380 \times x'''^2$
$C = 1 = 1.$	$27 =$	$1 \times x'''3$
	1916847	Subtrahend.

Whence $x = x' + x'' + x''' = 463$, the required Root.

This form will serve for all numeral equations, and with nearly the same labour; as for

EXAMPLE II.

$3x_3 + 2x^2 - 5x = v =$	242235792	near root $x' = 400$
$v' =$	192318000	$= 3x'3 + 2x'2 - 5x'$
By the Rule, $v - v' =$	49917792	Resolvend.
divisor		New equation $x'' = 30$
$9x'^2 + 4x' - 5 = 1441595 = A$	43247850	$= 1441595 \times x''$
$9x' + 2 = 3602 = B$	3241800	$= + 3602 \times x''^2$
$3 = 3 = C$	81000	$+ 3 \times x''^3 = v - v'$
v''	46570650	Subtrahend.
	3347142	Resolvend.

By the Rule,		$x''' = 2$
divisor		New Equation.
$9x''^2 + 7204x'' + 1441595 = 1665815$	3331630	$1665815 \times x'''$
$9x + 3602 = 3872$	15488	$+ 3872 \times x'''2$
$3 = 3$	24	$+ 3 \times x'''3 = v - v' - v''$
	3347142	Subtrahend.
Hence, $x = x' + x'' + x''' = 432.$	000000	

By dividing the original equation by x—432 = 0, the other roots may be found, but in this case they are imaginary. But, instead of thus approximating to a root by single figures, we can (after the root has been sufficiently diminished) find, by a Table of *Logarithms,* as many places of figures at one operation, as there are places of figures in the logarithms; as in the following

EXAMPLE III.

Suppose $x^3 - 2x = 5$. near Root $x' = 2$

then $x'^3 - 2x' = 4$

Resolvend 1

By the Rule,

divisor.

$$\left.\begin{array}{ll} 3x'^2 - 2 = & 10 \\ 3x' = & 6 \\ 1 = & 1 \end{array}\right\}$$

near Root $x = ,09$

gives $10x'' + 6x''^2 + x''^3 = 0,949329$ Subtrahend.

Resolvend $0,050671 = v$.

This being now sufficiently reduced we proceed thus:—

By the Rule,

New Equation.

$$\left.\begin{array}{lll} 3x''^2 + 12x'' + 10 & = 11,1043 & (A) \\ 3x'' + 6 & = 6,27 & = B \\ 1 & = 1 & = C \end{array}\right\}$$

$11,1043x''' + 6,27x'''^2 + x'''^3 = ,050671 = V$ and by reversion of series, $x''' = \frac{V}{A} - \frac{B}{A^3}V^2 + \&c.$

Then, by logarithms,

	$V = ,050671$	Log.	v	8.704760
	$A = 11,1043$	Log.	a	1.045492
v — a	$\frac{V}{A} = ,00456319$	Log.	d	7.659268
d — a =	$\frac{V}{A^2}$		c	6.613776
6,27 =	B		b	0.797268
b + c + d =	$\frac{B}{A^3}V^2 = ,00001176$			5.070312

$$\frac{V}{A} - \frac{B}{A^3}V = x''' = ,00455143. \text{ whence } x = 2.09455143$$

And if ,004551 be put for x''', in the above new equation, the value and root would be again reduced, so that we should obtain $x'''' = ,0000004815424$, and consequently the root $x = 2,0945514815424$, true to the last figure.